Step in Time: Modeling Twin Regulator Clock Pendulums with Coupled Virtual Spring Oscillators

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Abstract

Describes the imposition of spring characteristics on a motor, and the networking of several of these virtual torsion springs such that their oscillations lock in phase.

Keywords: coupled oscillators, spiral springs, phase locking, synchronization

Introduction

In 1656 Christiaan Huygens, a mathematician from the Netherlands, patented the first pendulum clock. This research was prompted by his work in astronomy, which requires accurate timekeeping. Huygens' further work on the topic of pendulum clocks led him to other applications, such as the determination of longitude while on sea voyages: If you know the time difference, you know how far east or west you have travelled from a last known position. Unfortunately, the motion of a ship on the open sea causes problems with a clock that depends on the oscillation of a pendulum, since the clock is being subjected to an external oscillating force – that of the rolling of the waves – of a frequency different from its own. This led Huygens to explore spiral springs as an alternate power source for these marine clocks. Like pendulums, springs oscillate at the same frequency regardless of amplitude.

While the history of clock oscillators is a fascinating subject in itself, the general trend is to move away from coupling and phase locking – one generally wants a clock to oscillate at a consistent frequency under any and all circumstances, and not adjust itself to whatever oscillations happen to be occurring nearby. However, this very phenomenon is essential in the mechanics behind "twin regulator" clocks(Fig 1) whose pendulums keep themselves in time by reacting to the movements of the other. One of the more famous builders of these twin regulators was Abraham-Louis Breguet, who lived from 1747 to 1823. The clocks he sold to French and English royalty still have good results even now.

Our goal is to take the idea of the coupled pendulums in twin regulator clocks and apply it to virtual spring oscillators coupled over ethernet. As the twin pendulums would keep themselves in time by force interactions as they sway to and fro, our virtual spring oscillators will synchronize themselves by comparing their velocities directly over the network.

Building a single second-order oscillator

The first task is to create a virtual spiral spring whose properties we can manipulate. The properties of electric motors allow us to easily make such a motor into a virtual coil spring such as one might find in an old watch. In a motor, the voltage is proportional to the angular velocity, while the current is proportional to the torque. By assuming these values will follow the characteristics of a spring and controlling the motor accordingly, we essentially end up with a wheel that thinks it is a spring.

A flywheel on a torsional or spiral spring is affected by two major forces; the force of the spring and the force of frictional damping, both of which will play a part in our virtual spring. By Hooke's law, the torque from a torsional spring is:

$$T_s = -k\theta$$

where k is the spring constant and θ is the position of the wheel in radians. The damping torque of friction

Figure 1: Twin Regulator: H101 “Resonance”

This device consists of two independent precision clocks, whose pendulums swing exactly opposite to eachother in a phase shift of 180 degrees at all times.

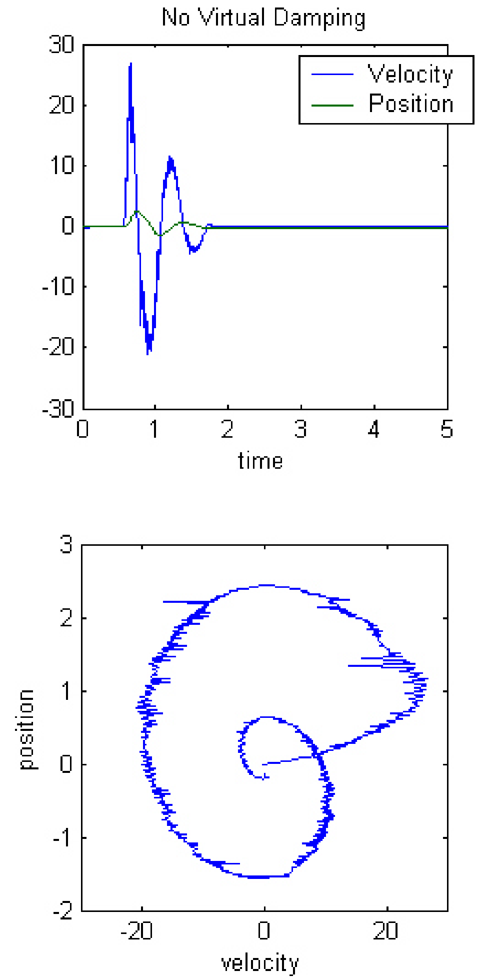

Figure 2: No Virtual Damping

No damping torque was added into the terms to power the motor. All damping is from friction inherently present in the system.

is proportional to the speed at which the wheel is turning, expressed as:

$$T_d = \alpha\omega$$

where α is some damping constant. By adjusting k and α, we can feed these two forces into the voltage going into the motor. The instantaneous velocity ω of the motor is obtained by reading in the voltage across the motor, subtracting off the extra voltage generated by the resistance in the coils of wire inside the motor, and multiplying the result by a constant based on the characteristics of the motor. In our experimental case, our motor was found to have a speed of 39 radians/sec (375 RPM) when powered by a 9 volt battery and under no load; making our motor constant 39/9 or about 4.03 $rad/v \cdot s$. The position of the flywheel θ is obtained by integrating the velocity of the motor as it turns the wheel. From these values for ω and θ we can determine the appropriate torque that a spring and frictional damping would apply to the wheel, convert these into volts back through the motor constant, and output back to the motor.

The trickiest part of the system is α, the damping constant. The spring constant can be chosen arbitrarily, but friction is already present in the system. We need a clockspring-like mechanism which oscillates with a long lifespan, so the key here is to chose

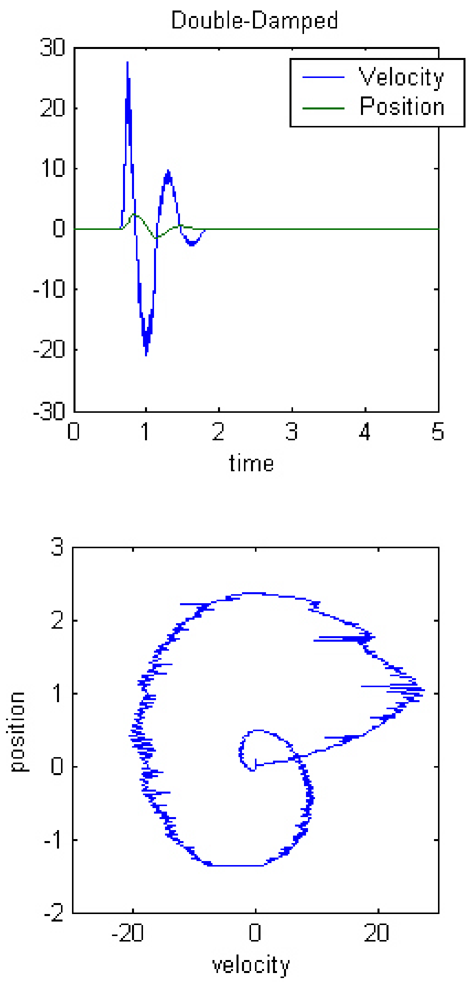

Figure 3: "Double" Damping

Damping constant α set equal to the natural damping of the system. This effectively damps the system twice; once virtually through the torques applied to the motor, and once naturally through the friction inherent to the system.

a damping constant which counteracts the friction already present in the system.

To examine how the damping constant affected the system, we performed several tests to see if we could determine the perfect damping constant. We began by taking data from the virtual spring with the damping constant equal to zero(Fig 2). We then adjusted the damping constant in a simulation until the empirical data and simulated data matched, in this way approximating the natural damping constant of the system. Three more tests were performed, one with the α set equal to this natural damping(Fig 3), a second with α set to the negative

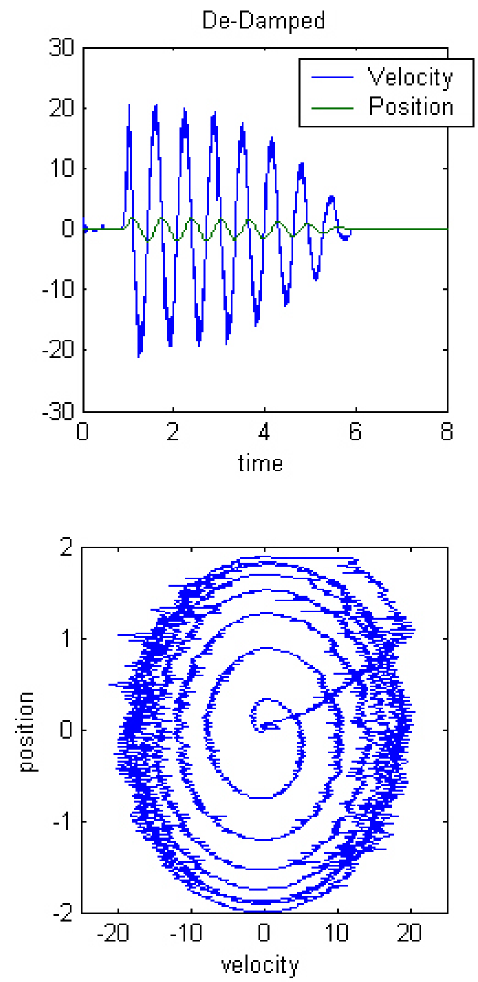

Figure 4: Dedamping

Damping set to negative the natural damping, in an attempt to counteract friction.

of the natural damping(Fig 4), and the third with α set to the negative of double the natural damping(Fig 5).

Upon comparing the results, we found that an α equal to the natural damping caused the spring to slow down slightly faster than naturally. An α equal to negative twice the natural damping added too much energy into the system, and the oscillations increased in amplitude until our sensors maxed out at 10 volts. If α was set to the negative of the natural damping, the spring stayed active for much longer than without damping, and did not start oscillating out of control. Unfortunately, our approximation for the natural damping of the system was not perfect. Upon adjusting this value to try and better it, we

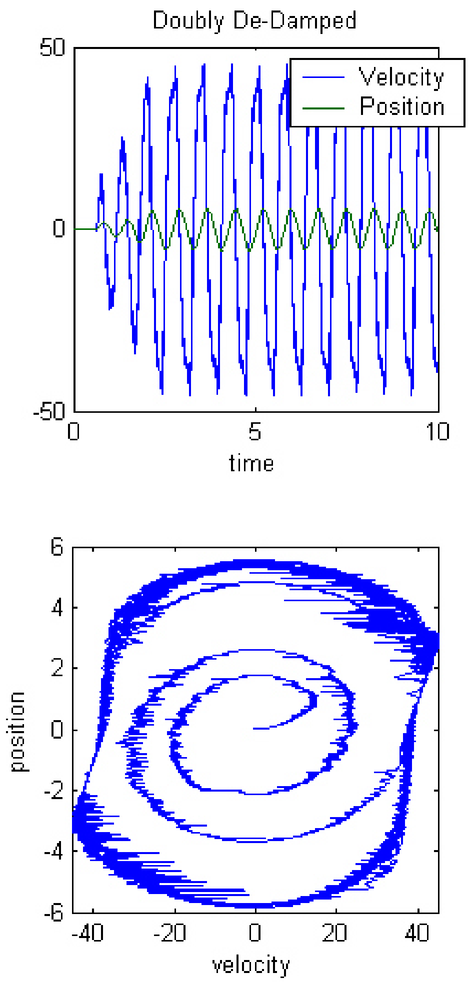

Figure 5: 2x Dedamping

Damping set to twice negative the natural damping, causing the oscillations to blow up over time.

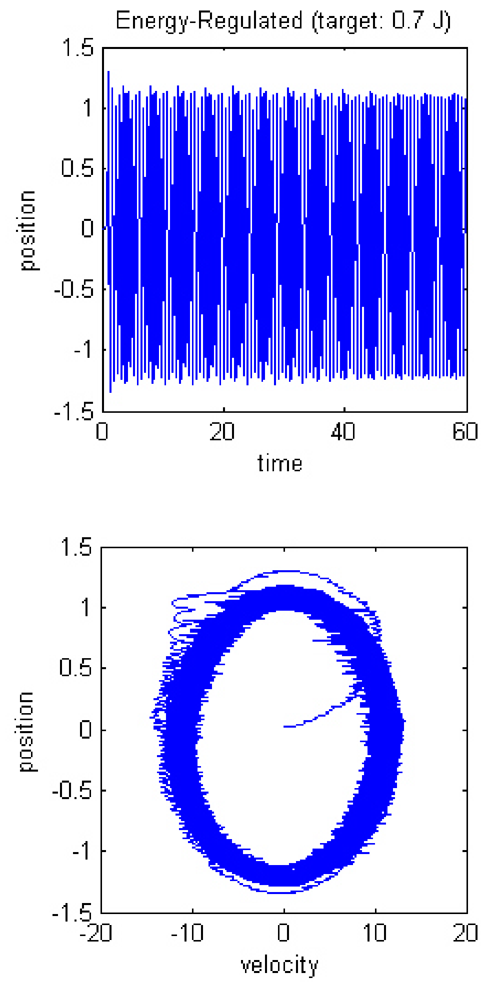

Figure 6: Energy-Regulated Damping

Damping set dynamically to keep the total mechanical energy of the system constant.

found that there were too many variables in the system to be able to pick a single value for α which would keep the spring going predictably. A more effective method of controlling our oscillating flywheel was needed.

The main problem is the presence of friction, be it virtual or natural. In physics, the Work-Energy Theorem states that if there are no nonconservative forces such as friction in a system, mechanical energy is conserved. This means that the sum of the Kinetic Energy and Potential Energy of the system is constant. By calculating the Kinetic and Potential Energies of our oscillator system and dynamically changing the value of α to keep the sum of these two values constant, it eliminates the unpredictable effects of other variables when α is set to a static value.

The Kinetic Energy(K) and Potential Energy(U) of a torsional spring are expressed as follows:

$$K = \frac{1}{2} I \omega^2$$

$$U = \frac{1}{2} k \theta^2$$

where k is the spring constant in N/rad^2, and I is the moment of inertia of the flywheel in $kg \cdot m$. Since k is defined and ω and θ came out of our data, we tweaked I until the maximum K and the maximum U calculated were approximately equivalent. The damping constant α was then set proportionally to

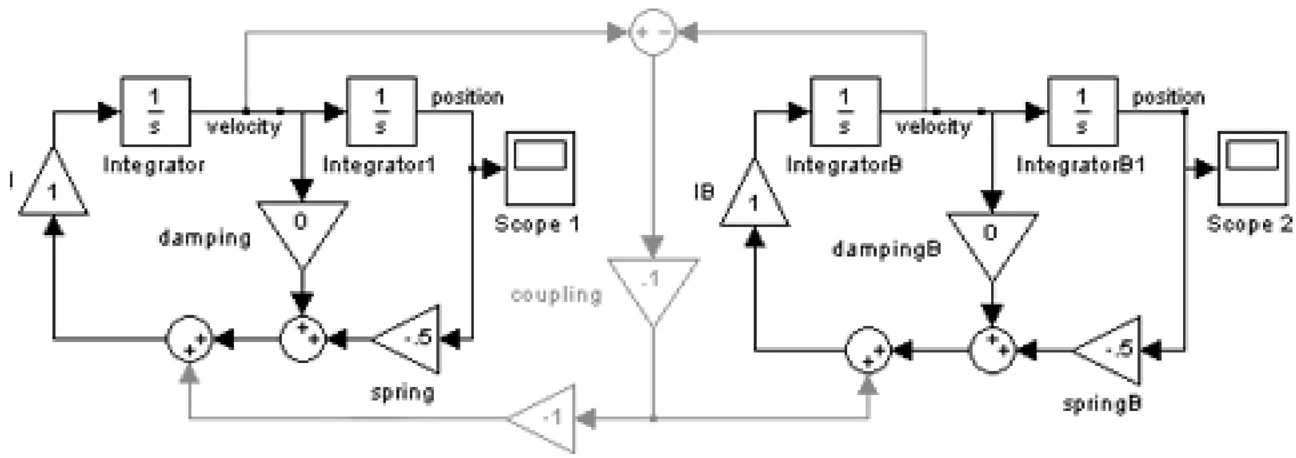

Figure 7: Simulation

Simulink diagram of the coupled oscillator system. As the diagram shows, the two oscillators are linked by a damper comparing their velocities, which adjusts the input to the motors accordingly.

the difference between the instantaneous total energy of the system and a defined total desired energy, such that when the energy was too low, virtual damping was decreased, and when energy became too high, damping was increased. In this way, we were able to get an oscillator which would operate predictably for an indefinite period of time. Data from this oscillator can be viewed in Figure 6.

Coupling of two oscillators

Once we have one oscillator and understand how dampers work, it is easy enough to link two of them together and dampen the difference between them. Just as we adjusted the input to the motor before based on the difference between the actual mechanical energy and the desired mechanical energy, coupling two oscillators adds another term to the input to the motor based on the difference between the two oscillators' instantaneous velocities. Each oscillator constantly broadcasts its angular velocity to the IP address of its partner, while simultaneously listening to the incoming velocity of the other oscillator. If it is going too slow, more energy is added; too fast, some is taken away. In a simulation of two coupled oscillators, synchronization was achieved in about five cycles(Fig. 8). The simulation was set up according to the diagram in Fig. 7.

Coupling of many oscillators

When an oscillator is trying to match more than one other oscillator on the network, a problem occurs where at one moment, oscillator A is trying to match the value sent from oscillator B, and at the next moment, trying to match the value sent from oscillator C, and at the next, trying to match the value sent from oscillator D. What is preferred is for oscillators A, B, C, and D to all try to match the average velocity of the group. We achieved this by determining the number of oscillators in the group to be coupled, averaging every n received velocity values, and using the result as the velocity-to-be-matched. In our largest test of the many-oscillator system, it took less than 3 minutes for 15 oscillators to synch up. Fig. 9 shows a scaled-down version of this test for visualization purposes.

Conclusion

We have proved that the coupling between two pendulums in a twin regulator clock can be mimicked using entirely virtual concepts applied to the control of electric motors. Instead of gravity controlling the sway of a pendulum, a computer can calculate the forces expected of a spiral spring and translate them into the movements of a motor. Replacing the effects of two pendulums each influencing the slight movement of their casing and thereby keeping eachother in step, we have created a network system

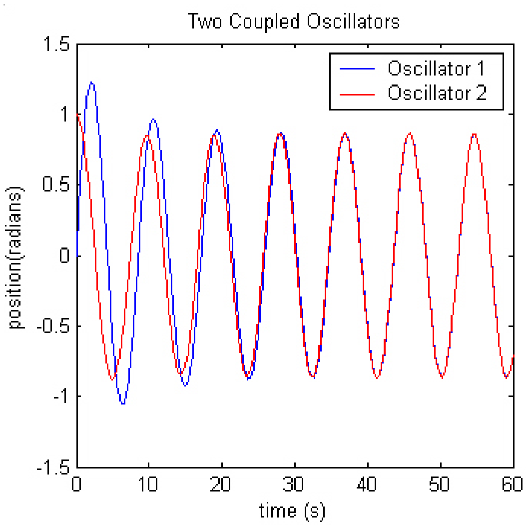

Figure 8: Coupled Oscillator Pair
Results of a simulated pair of coupled oscillators. Spring constant = 0.5, I=1, coupling proportionality constant = 0.1. Initial positions set $\approx \pi$ radians out of phase.

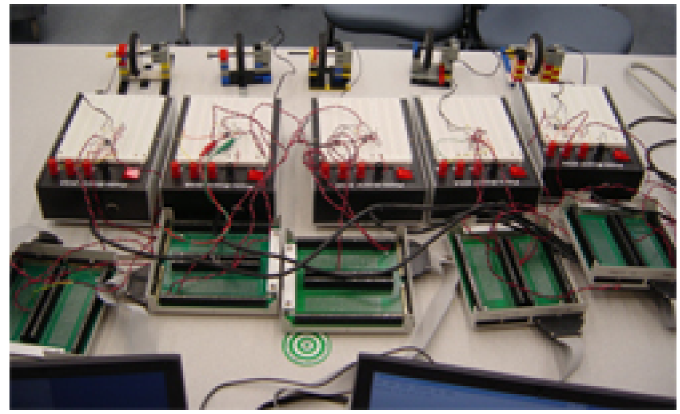

Figure 9: 5 Coupled Oscillators
Many-oscillators setup. At the top, five wheels are connected to motors, which go through an amplifier circuit before being fed through an analog→digital board and into laptops running the control program.

whereby each virtual spring communicates its velocity to the others, and have implemented a damper to effect changes into the motions of the motors. Essentially, we have recreated electronically the main concepts of one of the most accurate styles of clocks of the 1800s.

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Lab setup photos ruthlessly ganked off an anonymous folder on the Olin College Students' Public drive.